AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________

Forename(s) ______________________

Candidate signature ______________________

A-level MATHEMATICS

Unit Further Pure 3

Wednesday 17 May 2017 Morning Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

Answer **all** questions.

Answer each question in the space provided for that question.

1 It is given that $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where $$\mathrm{f}(x, y) = \frac{x + 2\sqrt{y}}{x + 1}$$

and $$y(1) = 4$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h\,\mathrm{f}(x_r, y_r)$$

with $h = 0.2$, to obtain an approximation to $y(1.2)$.

[2 marks]

(b) Use the formula

$$y_{r+1} = y_{r-1} + 2h\,\mathrm{f}(x_r, y_r)$$

with your answer to part **(a)**, to obtain an approximation to $y(1.4)$, giving your answer to three decimal places.

[3 marks]

QUESTION PART REFERENCE	**Answer space for question 1**

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QUESTION PART REFERENCE	Answer space for question 1

2 (a) It is given that $y = \mathrm{f}(x)$ is the solution of the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 5\frac{\mathrm{d}y}{\mathrm{d}x} + 6y = 3x^2 + 5$$

such that $\mathrm{f}(0) = 0$ and $\mathrm{f}'(0) = 1$.

(i) Without solving the differential equation, show that $\mathrm{f}'''(0) = -6$ and find the value of $\mathrm{f}^{(4)}(0)$.

[3 marks]

(ii) **Hence** find the first three non-zero terms in the expansion, in ascending powers of x, of $\mathrm{f}(x)$.

[2 marks]

(b) Find the general solution of the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 5\frac{\mathrm{d}y}{\mathrm{d}x} + 6y = 3x^2 + 5$$

[7 marks]

QUESTION PART REFERENCE	**Answer space for question 2**

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3 The first four non-zero terms in the expansion, in ascending powers of x, of $\ln(1+\sin x)$ are $x-\frac{1}{2}x^2+\frac{1}{6}x^3-\frac{1}{12}x^4$.

(a) (i) Write down the expansion of $\ln\left(1-\sin x\right)$ in ascending powers of x up to and including the term in x^4.

[1 mark]

(ii) **Hence** show that the first two non-zero terms in the expansion, in ascending powers of x, of $\ln(\cos x)$ are $-\frac{1}{2}x^2-\frac{1}{12}x^4$.

[3 marks]

(iii) Hence, or otherwise, find the first two non-zero terms in the expansion, in ascending powers of x, of $\ln(\sec x+\tan x)$.

[3 marks]

(b) Find $\lim\limits_{x\to 0}\left[\dfrac{\ln\left(\sec x+\tan x\right)}{2x+5x^3}\right]$.

[3 marks]

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4 (a) Given that $x = e^t$ and y is a function of x, show that $x^2 \dfrac{d^2y}{dx^2}$ can be expressed in the form $\dfrac{d^2y}{dt^2} + n\dfrac{dy}{dt}$, where n is an integer.

[5 marks]

(b) Hence use the substitution $x = e^t$ to find the general solution of the differential equation

$$x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = 10x, \quad x > 0$$

giving your answer in the form $y = f(x)$.

[5 marks]

QUESTION PART REFERENCE	Answer space for question 4

QUESTION PART REFERENCE	Answer space for question 4

5 Evaluate the improper integral $\int_0^{\frac{\pi}{6}} \left(\frac{2}{3x} - \frac{\sin 3x}{1 - \cos 3x} \right) \mathrm{d}x$, showing the limiting process used. Give your answer as a single term.

[8 marks]

QUESTION PART REFERENCE	**Answer space for question 5**

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6 At any point (x, y) on a curve C,

$$\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = 2(x-1)\,\mathrm{e}^{-2x} + 4$$

(a) By using an integrating factor, find the general solution of this differential equation, giving your answer in the form $y = \mathrm{f}(x)$.

[4 marks]

(b) Show that C has a horizontal asymptote and state the equation of this asymptote.

[2 marks]

(c) The curve C passes through the point $\left(-1,\ 2+4\mathrm{e}^2\right)$, and the line $y = k$ intersects C in three distinct points. Find all possible values for the constant k.

[7 marks]

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7 The Cartesian equation of a parabola C is $y^2 = 8(2 - x)$, $x \leqslant 2$.

(a) (i) Show that $y^2 = 8(2 - x)$ may be written in the form $x^2 + y^2 = (k - x)^2$, where k is an integer.

[1 mark]

(ii) Using the origin O as the pole and the positive x-axis as the initial line, show that, for $r \geqslant 2$, the polar equation of the parabola C is

$$r = 2\sec^2\frac{\theta}{2}, \quad -\pi < \theta < \pi$$

[4 marks]

(b) The straight line with polar equation $\tan\theta = \sqrt{3}$ intersects the parabola C at the points P and Q.

(i) Find the polar coordinates of P and Q.

[3 marks]

(ii) The area of the region bounded by the line segment PQ and the curve C is A_1.

The area of the circle with diameter PQ is A_2.

Show that $\dfrac{A_2}{A_1} = \pi\sqrt{3}$.

[9 marks]

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END OF QUESTIONS